Gray-level image reconstruction using Bessel-Fourier moments

C. Toxqui-Quitl, L. Gutiérrez-Lazcano, A. Padilla-Vivanco, and C. Camacho-Bello

Laboratorio de Óptica y Visión por Computadora. Universidad Politécnica de Tulancingo. Calle Ingenierías No. 100, Huapalcalco. Hidalgo. México.

ABSTRACT

In this work, we reconstruct discrete image functions by means Bessel-Fourier polynomials. To measure the image reconstruction we use the Normalized image reconstruction error between the input and reconstructed images. We show that, a good reconstruction performance is found to be available for gray-level images. The reconstruction algorithm is implemented using the first forty zeros of the Bessel functions of the first kind. Experimental results are presented.

Keywords: orthogonal moments, image reconstruction, Bessel functions.

1. INTRODUCTION

Moments have played an important role in the context of pattern recognition, image analysis, and machine vision applications in the last years [1-3]. In 1961, Hu [4] used geometric moments to generate a set of invariants; however the recovery of the image from these kind of moments has been found to be quite difficult, since they do not have the orthogonally property. Two decades after Hu, in 1980, Teague [5] proposed Zernike moments derived from the basis set of orthogonal Zernike polynomials. Zernike moments have been shown to be rotation invariant and robust to noise. Also, a relatively small set of Zernike moments can characterize effectively the global shape of a pattern. Other orthogonal moments are derived from the Legendre, Chebyshev, orthogonal Mellin-Fourier [6], Chebyshev-Fourier, [7] and the pseudo-Zernike polynomials [8]. In general, low order moments represent the global shape of an image and the high order moments the detail. Specifically, image reconstruction using orthogonal Fourier-Mellin moments (OFMM) and Chebyshev-Fourier moments (CHFM) have shown almost the same results and are superior to others moment sets in describing binary images [7]. Recently, a new set of orthogonal moments has been proposed from the Bessel-Fourier polynomials [9]. As stated by Bin X. et al [5], the Bessel radial polynomials have uniformly distributed zeros within the radial distance interval. It means that, these moments can describe with a very good accuracy the high-spatial frequency components, but newly they have been proven using practically binary images. However, for the case of images in gray scale, high and low spatial frequency components correspond, respectively, to sharp and smooth transitions of gray level intensities. These intensity transitions can be difficult to retrieve by means a certain kind of moments. In the recent literature has been analyzed several orthogonal moment sets for the reconstruction of binary objects as ideograms, characters, digits and so on; which are commonly used as test images in the pattern recognition areas. Typically, a lot of the image analysis has been done with this kind of objects. In this paper is analyzed the reconstruction performance by means the complex Bessel-Fourier orthogonal moments, using gray level images. From the point of view of the numerical effort, binary and gray scale images require different number of moments for their reconstruction; a few low orders are usually enough for the retrieval of binary images. However, the high order moments play a more important role for the reconstruction of gray level images [10-11]. In this work is shown that, the Bessel-Fourier moments give a good performance for the retrieval of gray scale images with a relatively low order of moments.

Our exposition is organized as follows: in section 2 is presented a general review of the Bessel-Fourier moments of an image based on the complex circular Bessel-Fourier polynomials. This review includes the orthogonallyproperty of the polynomial set. In Section 3, the discrete version of the moments and the reconstruction of some gray-level images are presented. The image reconstructions are measured with the normalized image reconstruction error (NIRE) in the section 4. Finally in Section 5, the conclusions to our research are presented.

22nd Congress of the International Commission for Optics: Light for the Development of the World,
edited by Ramón Rodríguez-Vera, Rufino Díaz-Uribe, Proc. of SPIE Vol. 8011, 80112T
2011 SPIE · CCC code: 0277-786X/11/$18 · doi: 10.1117/12.903391

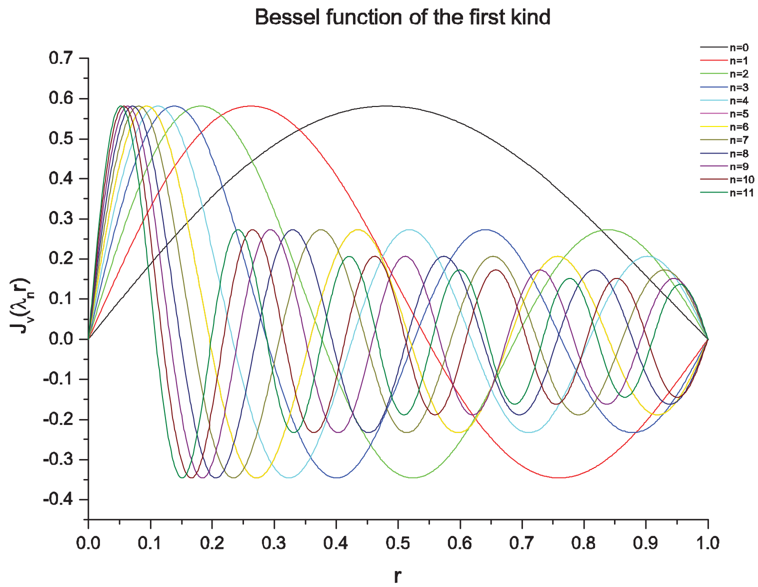

Figure 1. Bessel functions of the first kind for $n = 0, 1, 2, \ldots, 11$

2. BESSEL-FOURIER MOMENTS: A BRIEF REVIEW

Image Bessel-Fourier moments are a set of descriptors based on the Bessel functions of the first kind. A definition of these moments is given by

$$B_{n,l} = \frac{1}{2\pi a_n} \int_0^{2\pi} \int_0^1 f(r,\theta)\, J_\nu(\lambda_n r)\, e^{-il\theta} r dr d\theta, \tag{1}$$

where $n = 0, 1, 2, 3, \ldots$, and $l = 0, \pm 1, \pm 2, \pm 3, \ldots$are the radial and harmonic orders, $f(r,\theta)$ is the discrete image function, $J_\nu(r)\, e^{-il\theta}$ are the Bessel-Fourier polynomials, $a_n = \left[\frac{J_{\nu+1}(\lambda_n)}{2}\right]^2$ is a normalization factor, and λ_n are the zeros of the Bessel functions [12]. The radial polynomials satisfy the orthogonality property over the radial interval [0,1], which is given by

$$\int_0^1 r J_\nu(\lambda_n r)\, J_\nu(\lambda_k r)\, dr d\theta = a_n \delta_{nk}, \tag{2}$$

where δ_{nk} is the delta of Kronecker. The radial Bessel polynomials are defined by

$$J_\nu(r) = \sum_{k=o}^{\infty} \frac{(-1)^k}{k!\Gamma(\nu + k + 1)} \left(\frac{r}{2}\right)^{\nu+2k}, \tag{3}$$

where ν is a real constant, $\Gamma(\alpha)$ is the gamma function. Some curves of the radial polynomials for $\nu = 1$ and $\lambda_0 \ldots \lambda_{11}$ are shown in the Fig. 1.

3. DISCRETE IMAGE RECONSTRUCTION

The moments of an image are shape descriptors and to make possible the reconstruction, the first step is computed moments until a maximum order. This last step can be implemented by means the Eq. 1 in the discrete way.

Let $I(r_{u,v}, \theta_{u,v})$ be a discrete image function in polar coordinates with spatial dimensions MXN, their moments of order n with repetition l are given by

$$B_{n,l} = \sum_{u=o}^{M-1} \sum_{v=0}^{N-1} I(r_{u,v}, \theta_{u,v}) J_\nu(\lambda_n r_{u,v}) e^{-il\theta_{u,v}}, \tag{4}$$

where the discrete polar coordinates

$$r_{u,v} = \sqrt{x_u^2 + y_v^2}, \tag{5}$$

$$\theta_{u,v} = \arctan\left(\frac{y_v}{x_u}\right), \tag{6}$$

are transformed by

$$x_u = a + \frac{u(b-a)}{N-1}, \tag{7}$$

$$y_v = b - \frac{v(b-a)}{M-1}, \tag{8}$$

for $u = 0, \ldots, M-1$ and $v = 0, \ldots, N-1$. The real numbers a and b take values according to if the image function is mapped outside or inside an unit circle. The numbers $n = 0, \ldots, Max$ and $l = 0, \ldots, \pm Max$ are the highest orders selected for reconstructing an image and they are found to be image dependent. In this point, it can be obtained the Bessel-Fourier moments of a test image as shown in the Fig. 2.

Figure 2. A $200X200px$ gray-level test image

The reconstructed or retrieval image complex discrete distribution of the image is given by

$$I_R(r, \theta) = \sum_{n=0}^{Max} \sum_{l=0}^{Max} B_{n,l} J_\nu(\lambda_n r_{u,v}) e^{-il\theta_{u,v}}. \tag{9}$$

An real intensity distribution of the test image can be obtained by the modulus $|I_R(r, \theta)|$. Examples of the image reconstruction for several values of MAX are shown in the Figures 3, 4, and 5.

It is clear that, the advantadges in gray-level image retrieval are observed by using the Bessel-Fourier moments. This last is shown by the maximum order of reconstruction used in each case. If we compare with a more traditional method as Zernike moments, the convergence using the Bessel-Fourier polynomials is faster than with Zernike polynomials [10]. However it is evident that, a hole is formed in the image center. It is due to the amplitude of the polynomials when $r \to 0$.

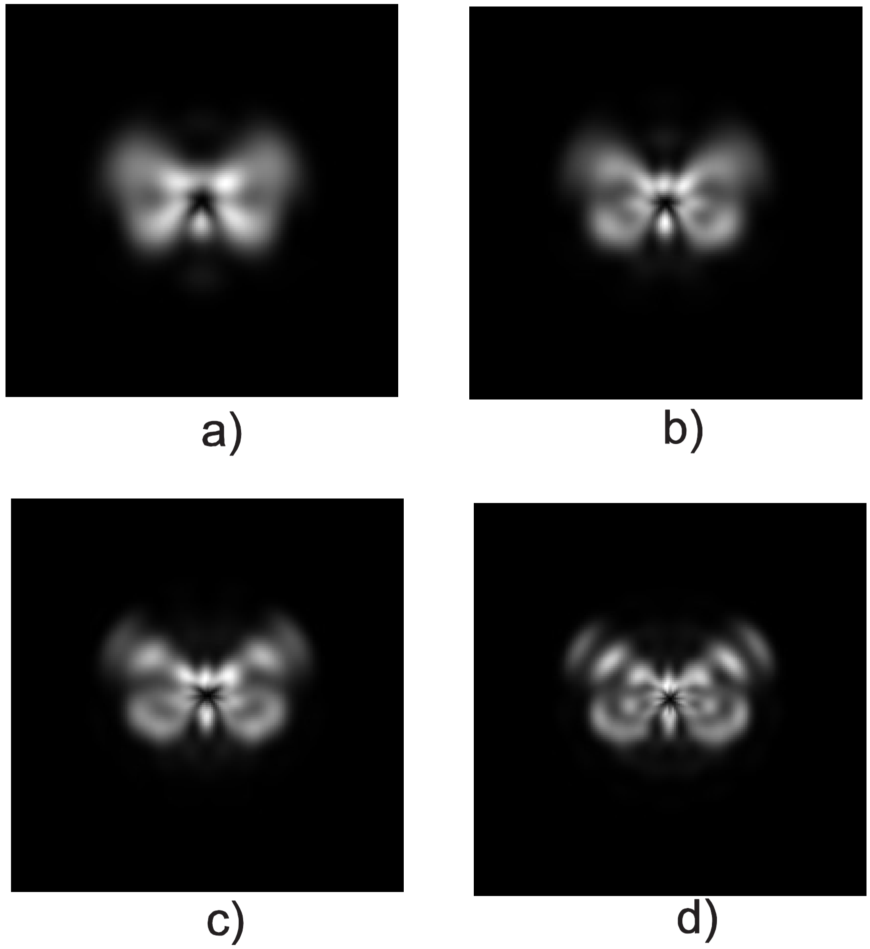

Figure 3. Image reconstruction, for $n = l = 0, \ldots, MAX$ (a) $MAX = 6$, (b) $MAX = 8$, (c) $MAX = 10$, (d) $MAX = 12$

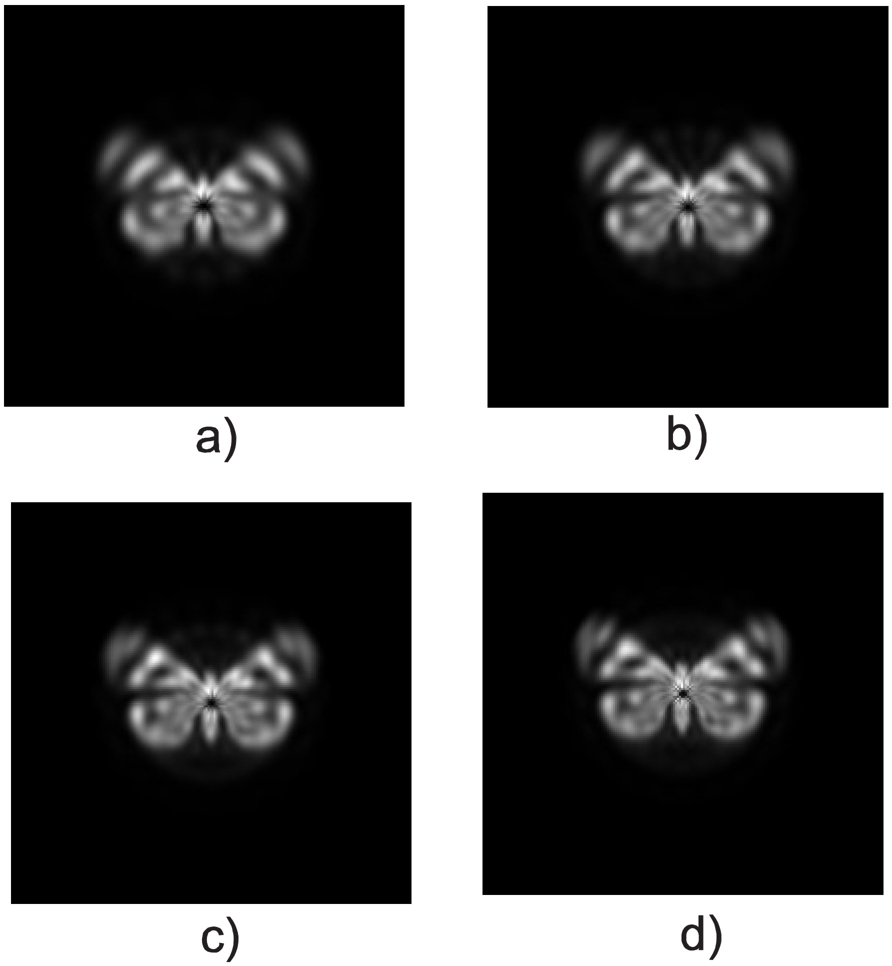

Figure 4. Image reconstruction, for $n = l = 0, \ldots, MAX$ (a) $MAX = 14$, (b) $MAX = 16$, (c) $MAX = 18$, (d) $MAX = 20$

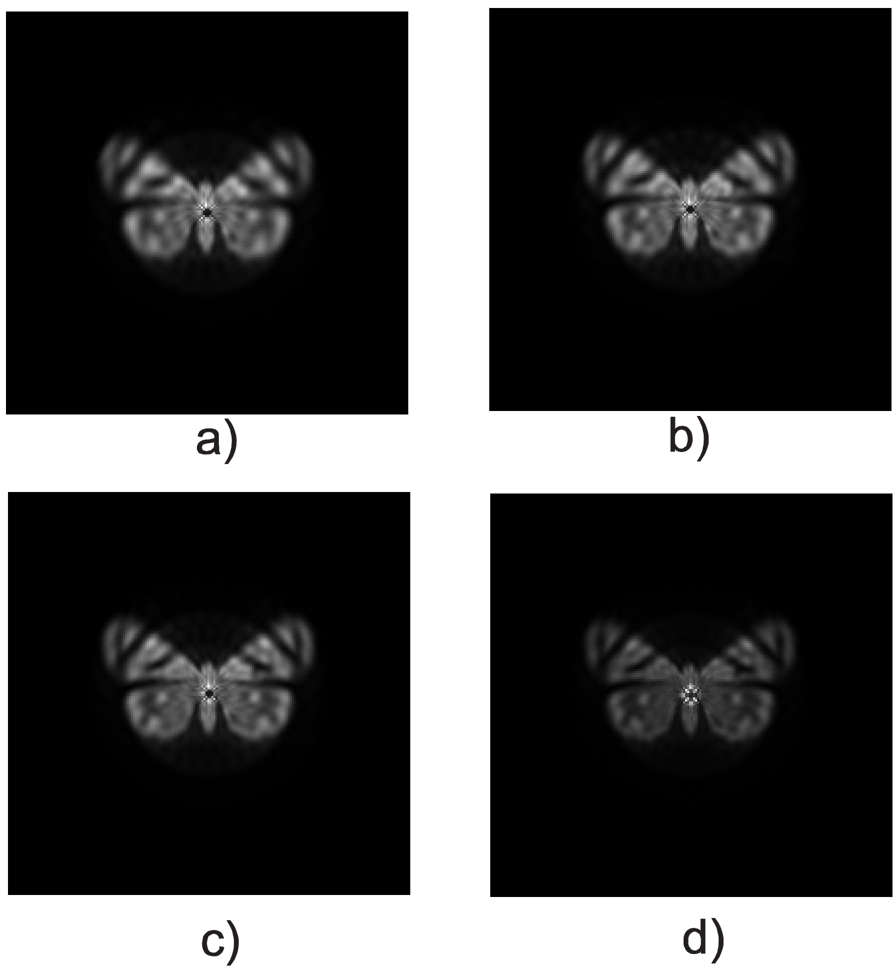

Figure 5. Image reconstruction, for $n = l = 0, \ldots, MAX$ (a) $MAX = 22$, (b) $MAX = 24$, (c) $MAX = 26$, (d) $MAX = 28$

Figure 6. Image reconstruction, for $n = l = 0, \ldots, MAX$ (a) $256X256$ Camera man image, (b) Reconstruction using $MAX = 8$, (c) $256X256$ Lena image, (d) Reconstruction using $MAX = 40$

Additionally, two test images are reconstructed using Bessel-Fourier moments, the results are shown in the Fig. 6.

Two examples more are reconstructed. These cases are the "cameraman" and "lena" images, using the Bessel-Fourier moments. As in the first case, the image intensity distributions of both are retrieved. This reconstruction is achieved by the maximum order $MAX = 40$. In the "lena" image the detail level reached is well enough to identify the scene. Using other kind of moments as Zernike, it is not possible to retrieve the same information as Bessel-Fourier does.

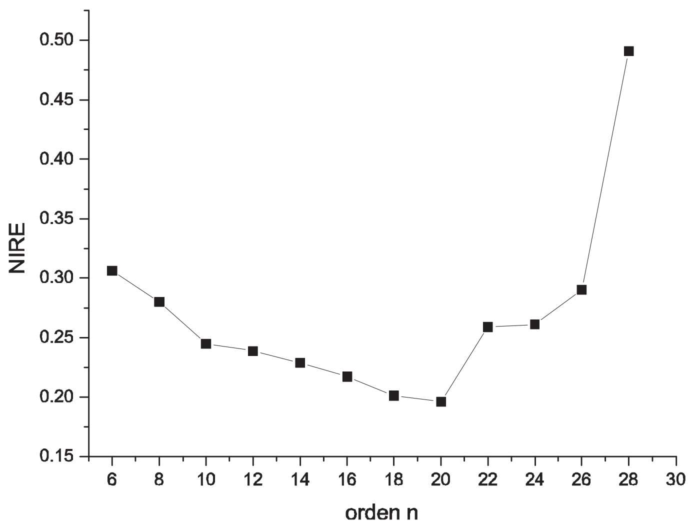

Figure 7. *NIRE* vs. reconstruction order

4. IMAGE RECONSTRUCTION ERROR

In order to test the reconstruction performance of the Bessel-Fourier moments, the $NIRE$ [13] between an input test image $I(u,v)$ and a reconstruction image I_R is implemented. It is defined by

$$NIRE = \frac{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\left[I\left(u,v\right)-I_R\left(u,v\right)\right]^2}{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\left[I(u,v)\right]^2} \tag{10}$$

In general, it is seen that the reconstruction of the Bessel-Fourier moments has a good performance in the three cases of the test images. Regardless of the different characteristics of each one. The values for the $NIRE$ in the butterfly image case are shown in the Fig. 7.

5. CONCLUSIONS

We have reconstructed three cases of gray-level images using the Bessel-Fourier moments. The NIRE is increased after the 20 order, as is shown in the Figure 7. It is due to the gaussian noise inherent in the image. For higher orders the NIRE can take oscillations. All the reconstructed cases are presenting a dark hole in the image center. This artifact tend to dissapear for the higher reconstruction orders. This behavior can be seen from the first image in the Fig. 3 until last one in the Fig. 5. In the cases of Lena and Cameraman images, the reconstruction is completed until $n = l = 40$ orders. The detail is better in the image center than in the boundaries as shown in the Fig. 6. In general we can conclude that the reconstruction performance is very adequate using Bessel-Fourier moments, it is based in the qualitative and quantitative results.

6. ACKNOWLEDGEMENTS

This work is supported by the project PROMEP No. UPTLGO-PTC-033

REFERENCES

1. *Pattern recognition with moment invariants-a comparative study and new results*, Pattern Recognition 24 (12), 1117-1138 (1991).
2. C. Kan, M. D. Srinath, *Invariant character recognition with Zernike and orthogonal Fourier-Mellin moments*, Pattern Recognition 35, 143-154 (2002).
3. R. J. Prokop and P. Reeves, *A survey of moment-based techniques for unoccluded object representation and recognition*, CV-GIP: Graph, Models Image Process 54, 438-460 (1992).
4. M.K. Hu, *Visual pattern recognition by moment invariants*, IRE. Trans. Inform. Theory 8, 179-187 (1962)
5. M. R. Teague, *Image analysis via the general theory of moments*, J. Opt. Soc. Am. 70, 920-930, (1980).
6. Y. L. Sheng and L. X. Shen, *Orthogonal Fourier-Mellin moments for invariant pattern recognition*, J. Opt. Soc. Am. A 11, 1748-1757 (1994).
7. Z. L. Ping, R. G. Wu, and Y. Sheng, *Image description with Chebyshev-Fourier moments*, J. Opt. Soc. Am. A 19, 1748-1754 (2002)
8. Mukundan R, Ramakrishnan K. R., *Moment functions in Image Analysis*, World Scientific, Singapore (1998)
9. X. Bin, M. Jiang-Feng, and Xuan W. *Image analysis by Bessel-Fourier moments*, Pattern Recognition 43, 2620-2629 (2010)
10. Padilla-Vivanco A., Urcid-Serrano G., Granados-Agustín F., and Cornejo-Rodriguez A., *Comparative analysis of pattern reconstruction using orthogonal moments*, Optical Engineering, 46 1, 017002-1 a 017002-15, (2007).
11. Padilla-Vivanco A., Toxqui-Quitl C., Báez-Rojas J., *Gray level image reconstruction using Jacobi-Fourier moments*, Optical and Digital Image Processing Proc. of SPIE 7000,(2008).
12. Watson G. N. *A Treatise on the Theory of Bessel Functions*, Cambridge Mathematical Library,(2009).
13. Yunlong Sheng and Lixin Shen *Orthogonal Fourier-Mellin moments for invariant pattern recognition*, J. Opt. Soc. Am. A/Vol.11, No. 6/June 1994.