Diffractive and sampling effects in Fourier holographic filters using spatial light modulators

A. Padilla-Vivanco, R. E. Farias-Díaz, C. Toxqui-Quitl and J. C. Valdiviezo-Navarro

Laboratorio de Óptica y Visión, Universidad Politécnica de Tulancingo,
Calle Ingenierías No. 100. Huapalcalco C. P. 43629, Tulancingo, Hidalgo, México

ABSTRACT

In this work, the analysis of an optical - digital system based on the Fourier transform hologram architecture is presented. We are interested in the diffractive effects in the Spatial Light Modulators as the sampling in the Fourier plane and the diffraction produced by squircle-geometry apertures of pixels. Also, a mathematical analysis is done in terms of the fringes visibility of the filters. Simulations and experimental results of the method are shown.

Keywords: Fourier holograms, Sampling Theorem, Spatial light modulator, squircle aperture

1. INTRODUCTION

Nowadays, the implementation of some holographic systems require from a spatial light modulator (SLM). Typically these kinds of systems have had two drawbacks; the first is the pixel geometry because the corners are approximately rectangular. It is a common task for LCDs to work with pixels with blunt corners.[1] The second disadvantage of the SLMs can be found when they are used as a diffractive optical device and coherent illumination. Many replicas of the diffraction patterns in both Fraunhoffer and Fresnel planes are presented from apertures digitally written in the SLM. This last fact is due to the inherent grid presented in the SLM. These drawbacks limit the contrast and diffraction efficiency of the optical holographic filters. In this work we mathematically review some of these effects using the Sampling theorem and the squircle function. Also, we propose expressions for the intensity distributions and visibilities of the fringes presented in the Fourier holographic filters. These last expressions have been computed taking into account the kind of apertures displayed in the SLM to produce the reference beam. Specifically, the point source in a SLM must be approximated from a rectangular pixel or a extended circle of a few pixels. This paper is organized as follows: in the second section is presented a theoretical review of the Sampling theorem for SLMs and the expression of the Fraunhoffer diffraction patterns of squircle apertures.[2] Also in the same section, some visibility relationships for Fourier-like holographic filters are computed by the cases of two kinds of reference sources, namely, rectangular pixel and extended circle. In section 3, some simulation of the diffractive effects are shown. Finally in the section 4, the conclusions of the work are presented.

2. A THEORETICAL REVIEW

This section mathematically describes the sampling and diffractive effects that produce a spatial light modulator from its inherent grid structure.

Further author information: Send correspondence to E-mail: alfonso.padilla@upt.edu.mx, Telephone: 55 775 755 8301

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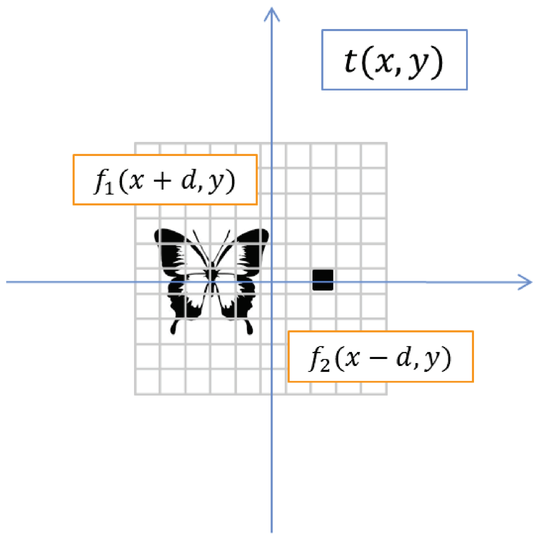

Figure 1. Sketch of the Input plane in a $2f$ coherent optical information processor. The grids in x and y directions represents the pixels of a SLM

2.1 Sampling theorem in a Spatial Light Modulator

Let $f_1(x, y)$ and $f_2(x, y)$ be two functions which are digitally written in a SLM. These functions are called, respectively, as object and reference

$$f(x, y) = f_1(x + d, y) + f_2(x - d, y). \tag{1}$$

Also the transmittance function of a SLM is given by

$$g(x, y) = \left[rect\left(\frac{x}{a}\right) rect\left(\frac{y}{b}\right)\right] * \left[\sum_{m}^{M} \delta(x - ml) \sum_{n}^{N} \delta(y - nl)\right], \tag{2}$$

where $*$ represents convolution. In this case, the pixel structure is represented by the rectangular functions along the x and y axis. So that, the general transmittance function is given by the product

$$t(x, y) = f(x, y)\, g(x, y). \tag{3}$$

A sketch of two functions digitally written in a SLM is shown in the Figure 1.

If the SLM is normally illuminated by an incident plane-wave field of amplitude A into an Fourier experimental setup, then the Fraunhoffer diffraction pattern can be obtained by the Fourier Transform $T = F\{t\}$ of the transmittance function t as follows

$$F\{t(x, y)\} = F\{f(x, y)\, g(x, y)\}, \tag{4}$$

and by means the convolution theorem, the last expression becomes

$$F\{t(x, y)\} = F\{f(x, y)\} * F\{g(x, y)\}. \tag{5}$$

Replacing Eq. (1) into Eq. (5) and developing terms

$$F\left\{t\left(x,y\right)\right\} = F\left\{f_1(x+d,y)+f_2(x-d,y)\right\} * F\left\{\left[rect\left(\frac{x}{a}\right)rect\left(\frac{y}{b}\right)\right] * \left[\sum_m^M \delta\left(x-ml\right)\sum_n^N \delta\left(y-nl\right)\right]\right\}, \quad (6)$$

and appliying the Fourier transform convolution theorem

$$F\left\{t\left(x,y\right)\right\} = \left[F\left\{f_1(x+d,y)\right\}+F\left\{f_2(x-d,y)\right\}\right] * \left[ab\sin c\left(au\right)\sin c\left(bv\right)\sum_m^M \delta\left(\frac{u-m}{l}\right)\sum_n^N \delta\left(\frac{v-n}{l}\right)\right], \quad (7)$$

$$F\left\{t\left(x,y\right)\right\} = \left[F_1\left(u,v\right)e^{-i2\pi(ud)}+F_2\left(u,v\right)e^{i2\pi(ud)}\right] * \left[ab\sum_m^M \sin c\left(au\right)\delta\left(\frac{u-m}{l}\right)\sum_n^N \sin c\left(bv\right)\delta\left(\frac{v-n}{l}\right)\right], \quad (8)$$

$$F\left\{t\left(x,y\right)\right\} = \left[F_1\left(u,v\right)e^{-i2\pi(ud)}+F_2\left(u,v\right)e^{i2\pi(ud)}\right] * \left[ab\sum_m^M \sin c\left(\frac{am}{l}\right)\delta\left(\frac{u-m}{l}\right)\sum_n^N \sin c\left(\frac{bn}{l}\right)\delta\left(\frac{v-n}{l}\right)\right] \quad (9)$$

Therefore, the Fourier transform of the function to be displayed into the SLM

$$\begin{aligned} T\left(u,v\right) &= ab\sum_{m,n}^{M,N}\sin c\left(\frac{am}{l}\right)\sin c\left(\frac{bn}{l}\right)F_1\left(\frac{u-m}{l},\frac{v-n}{l}\right)e^{-i2\pi d\left(\frac{u-m}{l}\right)} \\ &+ ab\sum_{m,n}^{M,N}\sin c\left(\frac{am}{l}\right)\sin c\left(\frac{bn}{l}\right)F_2\left(\frac{u-m}{l},\frac{v-n}{l}\right)e^{i2\pi d\left(\frac{u-m}{l}\right)}. \end{aligned} \quad (10)$$

Computing the intensity distribution from Fourier Transform by means $I(u,v) = \left|T\left(u,v\right)\right|^2$, as follows

$$\begin{aligned} I\left(u,v\right) &= a^2b^2\sum_{m,n}^{M,N}\sin c^2\left(\frac{am}{l}\right)\sin c^2\left(\frac{bn}{l}\right)\left[F_1^2\left(\frac{u-m}{l},\frac{v-n}{l}\right)\right. \\ &+ 2F_1\left(\frac{u-m}{l},\frac{v-n}{l}\right)F_2\left(\frac{u-m}{l},\frac{v-n}{l}\right)\cos\left(4\pi a\left(\frac{u-m}{l}\right)\right) \\ &+ \left.F_2^2\left(\frac{u-m}{l},\frac{v-n}{l}\right)\right]. \end{aligned} \quad (11)$$

This last expression represents MxN replicas of the Fraunhoffer diffraction patterns in the Fourier plane of the transmittance function which is written in the SLM.

2.2 Diffraction patterns of a squircle function

Fernandez Guasti et al,[2] introduce the analytical expression for the Fourier transform of the combination between the aperture of a circle and a rectangle, it now is called as squircle function. Its Fraunhoffer diffraction pattern can be computed by

$$\mathbf{G}\left(\rho,\phi\right) = \mathbf{2}\int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{M_s\left(\theta\right)sen\left[M_s\left(\theta\right)\rho k\cos\left(\theta-\phi\right)\right]}{\rho\cos\left(\theta-\phi\right)}\mathbf{d}\theta + 2\int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{\cos\left[M_s\left(\theta\right)\rho k\cos\left(\theta-\phi\right)\right]-1}{\rho^2\cos^2\left(\theta-\phi\right)}d\theta. \quad (12)$$

$$M_s(\theta) = \left\{ 2\frac{\left\{1-\left[1-s^2 sen^2(2\theta)\right]^{1/2}\right\}}{s^2 sen^2(2\theta)} \right\}^{1/2}. \tag{13}$$

this function is bounded such as $1 \leq M_s(\theta) \leq \sqrt{2}$.

2.3 Visibility of a holographic filter

In this part we are interested in computing expressions for the visibility of the Fourier holographic filters. The distribution intensity of the filter depends from the aperture shapes of the object and reference. Particularly, when the filter have been recorded using different geometries for the reference apertures.

2.3.1 Case I. Rectangular aperture and point source

We assume a transmittance function composed by a rectangular aperture of width $2a$ along with a point source represented by $k\delta(x-d)$; both are placed in the input plane, respectively, in the cartesian positions of d and $-d$, and if the transmittance function is normally illuminated by an incident plane wave field, then the Fraunhoffer pattern distribution intensity is given by

$$I(u,v) = k^2 + 4a^2 \sin c^2\left(\frac{2\pi au}{f}\right) + 2ak\sin c\left(\frac{2\pi au}{f}\right)\cos\left(\frac{2\pi(2d)u}{\lambda f}\right), \tag{14}$$

where λ and f are respectively the wavelength of the beam and the focal length of the lens used in the optical information processor. The visibility is as follows

$$V(u,v) = \frac{2\left|aksinc\left(\frac{2\pi au}{\lambda f}\right)\right||k|}{4a^2 sinc^2\left(\frac{2\pi au}{\lambda f}\right)+k^2}. \tag{15}$$

2.3.2 Case II. Extended aperture and circular extended source

As a second example we assume a transmittance function composed by an arbitrary aperture $g(x,y)$ and a circular extended source of radius a, both are place respectively in the cartesian positions of d and $-d$. Then the intensity distribution can be computed by

$$I(u,v,a,\rho) = |G(u,v)|^2 + \frac{J_1^2\left(\frac{2\pi a\rho}{\lambda f}\right)}{\left(\frac{2\pi a\rho}{\lambda f}\right)} + 2|G(u,v)|\left|\frac{J_1\left(\frac{2\pi a\rho}{\lambda f}\right)}{\left(\frac{2\pi a\rho}{\lambda f}\right)}\right|\cos\left(\frac{2\pi a\rho}{\lambda f}\right), \tag{16}$$

where $G(u,v) = F[g(x,y)]$ and the visibility is expressed by

$$V(u,v) = \frac{2|G(u,v)|\left|\frac{J_1\left(\frac{2\pi a\rho}{\lambda f}\right)}{\left(\frac{2\pi a\rho}{\lambda f}\right)}\right|}{|G(u,v)|^2 + \frac{J_1^2\left(\frac{2\pi a\rho}{\lambda f}\right)}{\left(\frac{2\pi a\rho}{\lambda f}\right)^2}}. \tag{17}$$

3. SIMULATION AND EXPERIMENTAL SETUP

As we mentioned before, two kinds of problems are presented when a SLM is used as transmittance function into an optical coherent processor, which is used as Fourier holographic system. The first problem are the replicas presented in the Fourier plane of the system of the Fraunhoffer diffraction pattern of the transmittance function, and the second inconvenient are the diffractive effects due to the pixels with blunt corners. An experimental example of the replicas is shown in the Figure 2 where is presented the intensity retrieval of an object and its conjugate from a holographic filter. In the Figure 3, a simulation of different squircles are shown along with their respective diffraction patterns. A sketch of the experimental setup is shown in the Figure 4,

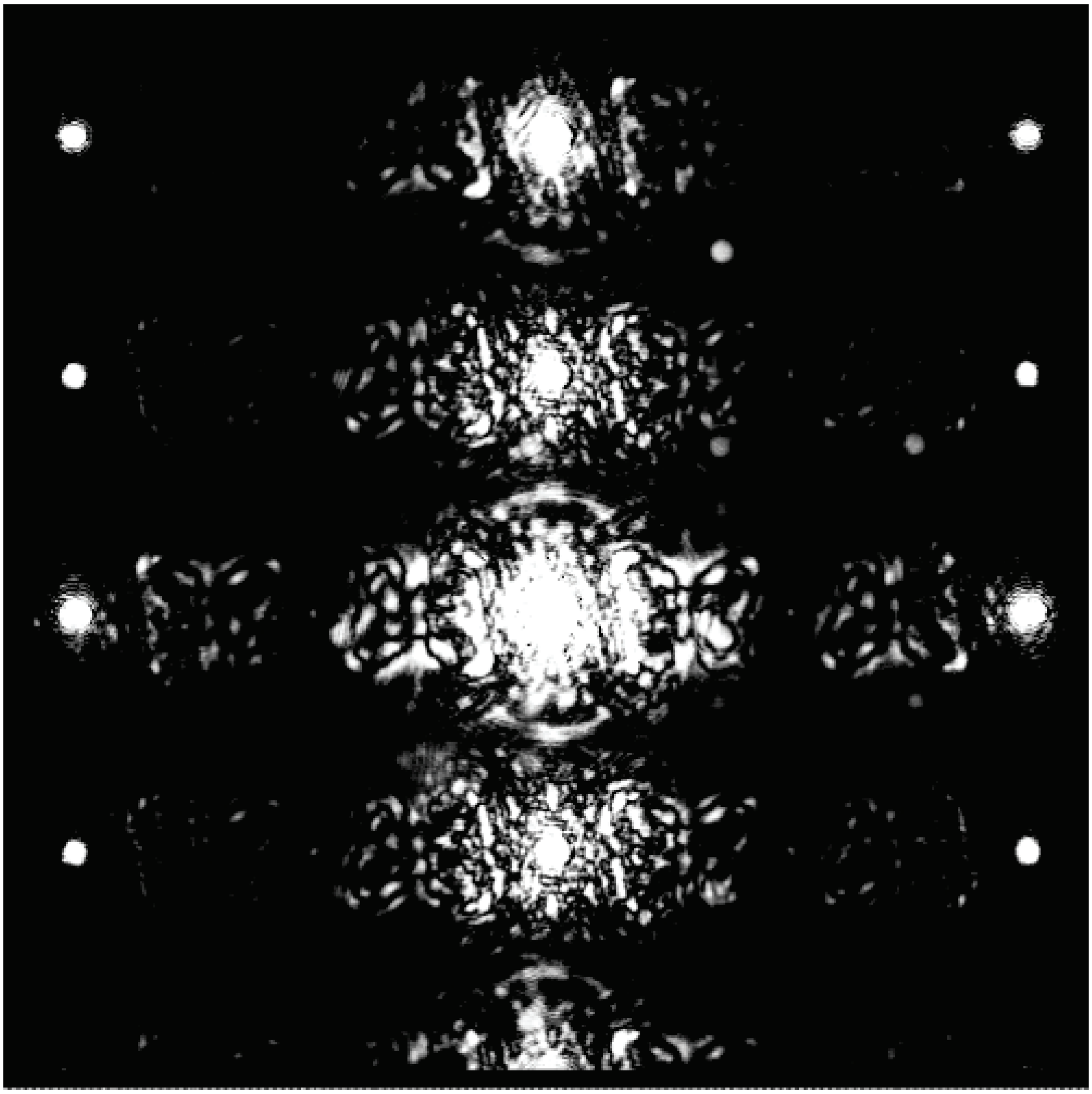

Figure 2. Replicas of the Fraunhoffer diffraction patterns from an holographic filter previously displayed in a SLM, which is illuminated with coherent light

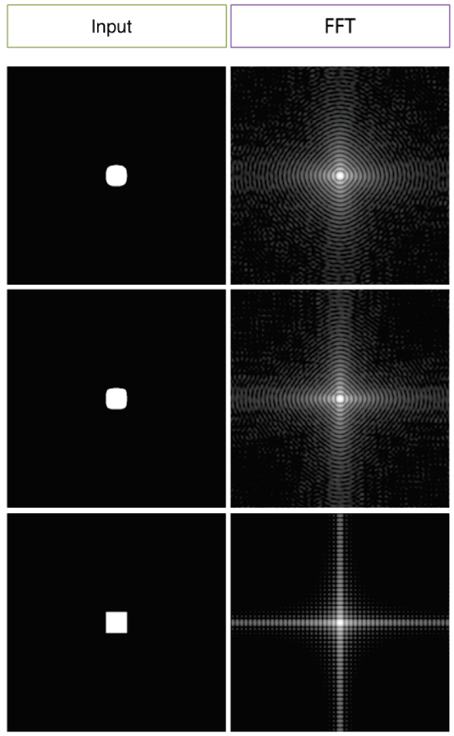

Figure 3. Fraunhoffer Diffraction patterns of pixels with blunt corners described by the squircle function

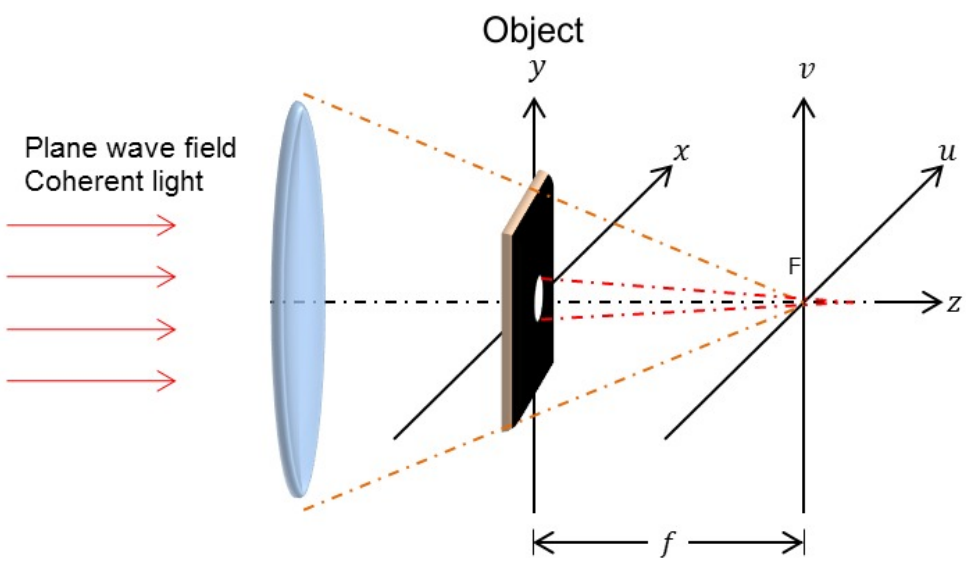

Figure 4. Sketch of the experimental setup to obtain the Fourier transform using a SLM as transmittance function with blunt pixels

Figure 5. Object and its conjugate displayed using the Gamma correction operation

4. CONCLUSIONS

We have presented a mathematical review of some diffractive effects of SLMs, when they are used as transmittance functions into optical coherent information processors. Particularly, in the case of using them for recording of in-line holographic filters. According with the computation of the visibility expression for the holographic filters, they can be affected by the geometry shape of the reference source, which is commonly used to generate the reference beam. The shape of the reference source in a Fourier holographic system which is implemented in a SLM can be diversified in concordance with the number of pixels taken into account. As well as the pixel structure and the corner effects. In the pixel simulation presented here, a structure that is a combination between a circle and rectangle can generate different Fraunhoffer diffraction patterns and consequently of this, it can change the intensity distribution of the holographic filter. The main conclusion of this work is that, the diffractive and sampling effects produce different intensity distributions in holographic filters, affecting the contrast or visibility and the diffraction efficiency of the filters.

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