Optimization of wavefront coding imaging system using heuristic algorithms

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ABSTRACT

Wavefront Coding (WFC) systems make use of an aspheric Phase-Mask (PM) and digital image processing to extend the Depth of Field (EDoF) of computational imaging systems. For years, several kinds of PM have been designed to produce a point spread function (PSF) near defocus-invariant. In this paper, the optimization of the phase deviation parameter is done by means of genetic algorithms (GAs). In this, the merit function minimizes the mean square error (MSE) between the diffraction limited Modulated Transfer Function (MTF) and the MTF of the system that is wavefront coded with different misfocus. WFC systems were simulated using the cubic, trefoil, and 4 Zernike polynomials phase-masks. Numerical results show defocus invariance aberration in all cases. Nevertheless, the best results are obtained by using the trefoil phase-mask, because the decoded image is almost free of artifacts.

Keywords: Wavefront encoding, Computational imaging, Genetic algorithms.

1. INTRODUCTION

Wavefront Coding (*WFC*) was first proposed by Dowsky and Cathey in 1995. This system makes use of an aspheric Phase Mask (PM) and digital image processing to extend the Depth of Field (*EDoF*) of computational imaging systems[1]. At the same time, it allows reducing the size, weight and cost of imaging systems[2,3]. In traditional optical systems, *EDoF* is to reduce the aperture stop. In this way, the system presents less aberrations. Nevertheless, a smaller aperture increases the role of the diffraction, reducing the resolution and the amount of light that can be gathered by the system [4,5]. Extending the depth of field of incoherent optical system has been an active research topic in the last years. Moreover, this technique is based in the use of a wavefront coding element which general is nonabsorbing phase element placed at the aperture stop. Such that the *PSF* of an optical system is near invariant to focus-related aberrations [5,6]. However, the encoded images, captured by a digital detector array are blurred and they must be digitally filtered to remove the coding of the *PSF*. The image with diffraction-limited quality is obtained after performing the digital image processing with the deconvolution filter[7]. In the literature it is mentioned that in *WFC* systems the magnitude of the defocused *MTFs* have an evident fall compared whit the diffracted- limited *MTF*, this results in a poor quality coded image. Nevertheless, also shows the pass band appears to be zero-free spatial frequencies. That is important, because is possible recovered the image without loss in spatial frequency information compared with a traditional in-focus system [7, 12, 13]. The performance of *WFC* systems depends on design suitable phase profile to generate a near invariant defocus system. Without a proper design and an optimal value of the phase-mask strength, the technique would not be able to carry out defocus invariance. Some applications of *WFC* have been studied for: corrections of optical aberrations[8], microscopy[9], for obtaining high resolution images of the human retina[10], among other possible applications. This paper is organized as follows: section 2, A theorical review of phase-masks is done. In Section 3 the optimization of the phase deviation parameter is done. Finally, the results and conclusions are presented in Section 4.

2. PHASE-MASKS

The first phase-mask proposed to *EDoF* corresponds the cubic-phase-mask with rectangular aperture in 1993. The mathematical expression of the phase-mask in the exit pupil is determined by,

$$P(x,y) = rect(x,y)e^{i\alpha\left(x^3+y^3\right)}, \tag{1}$$

where α is the phase deviation parameter or strength of mask, measured in lambda units.

Current Developments in Lens Design and Optical Engineering XVIII, edited by R. Barry Johnson,
Virendra N. Mahajan, Simon Thibault, Proc. of SPIE Vol. 10375, 103750Y

A cubic phase-mask with a circular aperture is described as,

$$P_{fc}(x,y) = circ\left(\sqrt{\frac{x^2+y^2}{f_o}}\right)e^{i\alpha\left(x^3+y^3\right)}, \tag{2}$$

where f_o is the cutoff frequency system, given by,

$$f_o = \frac{r}{\lambda z}, \tag{3}$$

r is the radio of the circular aperture and z is the distance to the image plane.

The phase-mask based in the Zernike polynomial $Z_{3.3}(\rho,\theta)$ known trefoil on the circular pupil is given for,

$$P_{zt}(x,y) = circ\left(\sqrt{\frac{x^2+y^2}{f_o}}\right)e^{i\alpha Z_{3,3}(\rho,\theta)}, \tag{4}$$

where the expression of polynomial is,

$$Z_{3.3}(\rho,\theta) = \sqrt{8}\rho^3 \cos 3\theta , \tag{5}$$

with $\rho = \left(x^2 + y^2\right)^{1/2}$ and $\theta = \tan^{-1}(y/x)$.

Palillero et al., proposed the use of the sum of the Zernike polynomials Z_7, Z_8, Z_9, Z_{10}. According to the arrangement proposed by Mahajan[22], they correspond with the aberrations of primary coma and trefoil, in both directions (x and y)[12].

$$Z_{tot} = Z_7 + Z_8 + Z_9 + Z_{10}. \tag{6}$$

The expression that describe the phase-mask on the circular pupil is,

$$P_{sz}(x,y) = circ\left(\sqrt{\frac{x^2+y^2}{f_o}}\right)e^{i\alpha Z_{tot}(\rho,\theta)}. \tag{7}$$

The *PSF* of the *WFC* system is,

$$PSF(x,y) = \left\| FT\left\{p(x,y)\cdot e^{ikW_{20}\left(x^2+y^2\right)}\cdot e^{ik\alpha PM}\right\}\right\|^2, \tag{8}$$

FT is the Fourier transform operator., $p(x,y)$ defines the shape and size of the exit pupil. *PM* is the phase-mask function, for example, cubic, trefoil, or the sum of four Zernike polynomials. W_{20} is the typically defocus term. Moreover, the Optical Transfer Function (*OTF*) and *MTF* are given by,

$$OTF(u,v) = \frac{FT\{PSF\}}{\max\left[FT\{PSF\}\right]}, \tag{9}$$

$$MTF(u,v) = \left\| OTF(u,v)\right\|. \tag{10}$$

Figure 1(a) is a contour plot of the phase-masks. In Fig. 1(b) can be seen the 3D surfaces of the phase-masks, and in Fig. 1(c) is shown the *PSFs* of the phase-masks. Figure 2 shows the magnitude of defocused *MTFs* for a conventional optical system.

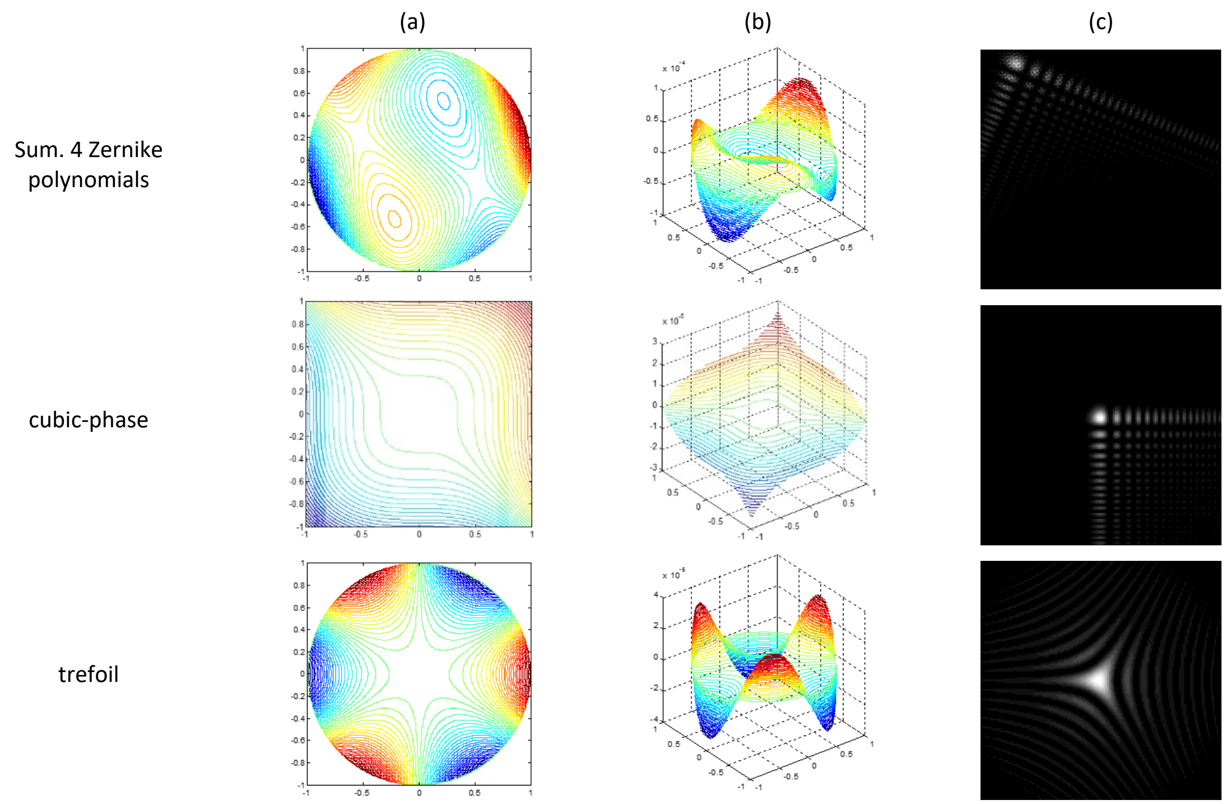


Figure 1. Surface of the phase-masks. (a) contour plot, (b) 3D plot and (c) PSF.

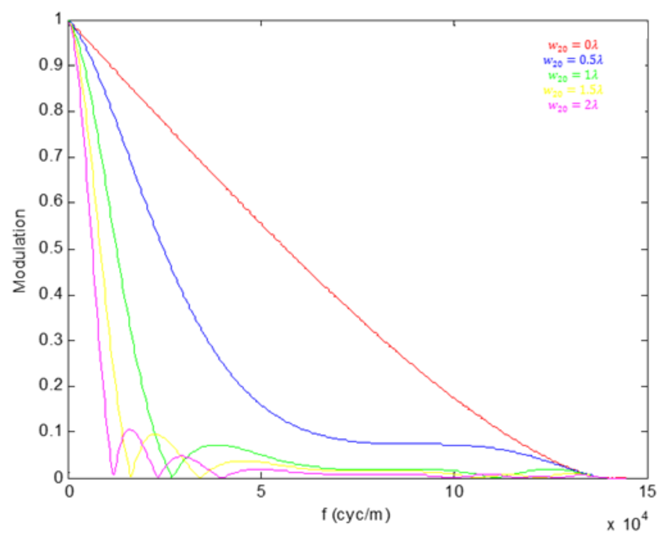


Figure 2: Magnitude of in focus MTF and defocus MTF in a traditional optical system.

3. PHASE OPTIMIZATION

The optimization of α is done by means of the Mean Square Error (*MSE*) between the in-focus *MTF* and the defocused *MTF*. The optimal phase deviation parameter α ranges from $\alpha \in [0-32]$. The specifications of the optical system for the phase optimization is described in the Table 1.

Table 1. Optical system specifications

Object distance	$608mm$
Image distance	$316mm$
Lens focal length	$208mm$
Lens refractive index	$n = 1.515$
Exit pupil diameter	$Dxp = 34mm$
Wavelength	$633nm$
CCD sensor size	$3.53mm \times 3.53mm$
Number of pixels	1024×1024
Pixel size	$3.45um$

A genetic algorithm contains a well-defined structure; an initial population of *n* individuals is generated. These individuals will be the α values obtained randomly within the limited search range. The objective function is given by the minimum value as,

$$Apt_{\alpha} = \min{}_{\alpha} \{MSE(\alpha)\} \tag{11}$$

The smallest value, the crossing and mutation is carried out in order to propose a new generation of individuals from the parameter α considered as Apt. With the new population generated the same procedure is carried out. After a certain number of iterations, the value of α converges towards the optimal value. Table 2 shows the optimal values for the phase deviation parameter α of each phase-mask.

Table 2. Optimal deviation parameters

Phase-mask	**Phase Deviation Parameter (strength)** α
Sum. 4 Zernike polynomials	19λ
Cubic-phase	20λ
trefoil	18λ

Encoded images, captured by a digital detector blurred due to the addition of the phase-mask aberration. They are digital filtered with a deconvolution filter. The image restoration process is achieved using the parametric Wiener filter. Where the value of *k* was interactively adjusted to provide a good visual result. In addition, a Butterworth low pass filter is used in order to obtain a high quality image.

The defocused *MTFs* corresponding to different defocus parameters, with strength obtained from Table 2, are shown in Figure 3. In all cases the curves maintain some similarity to the presence of defocus.

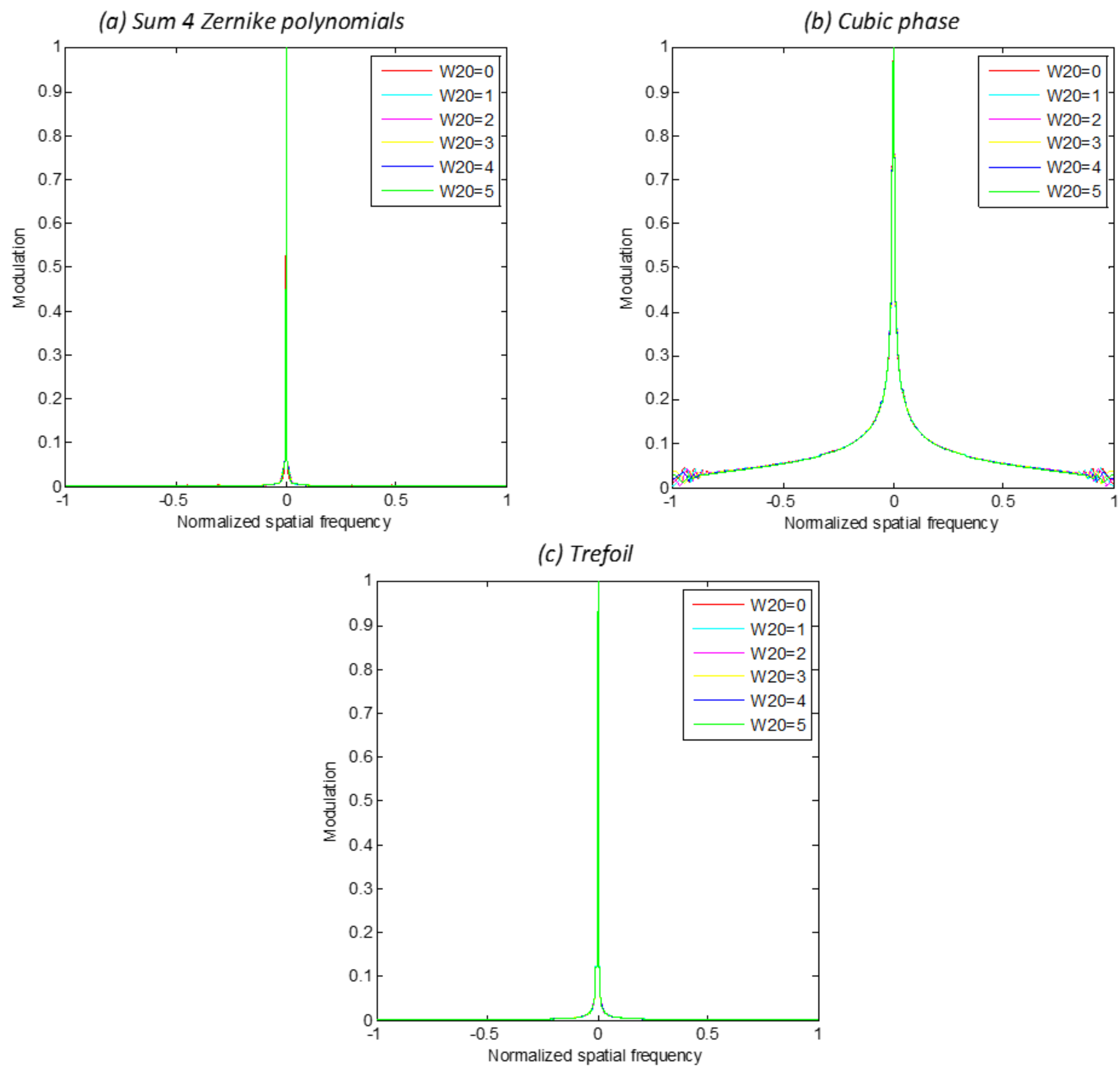


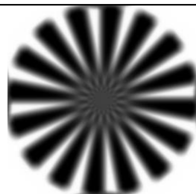
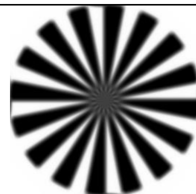
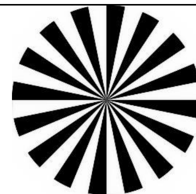
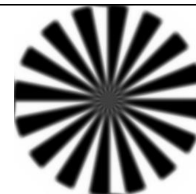
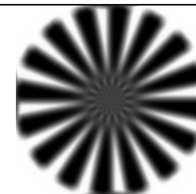
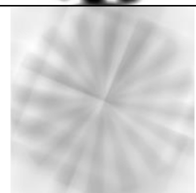
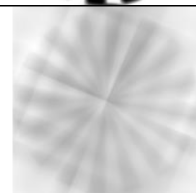
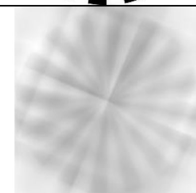
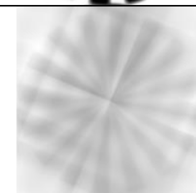
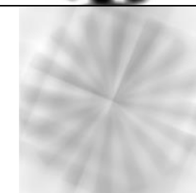
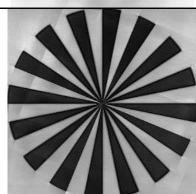
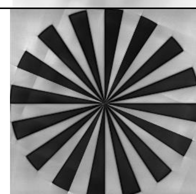
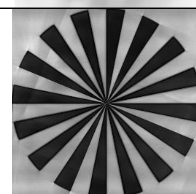
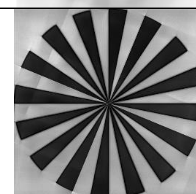
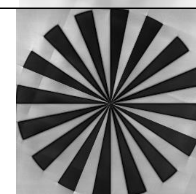
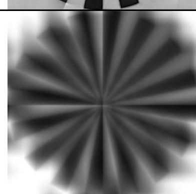
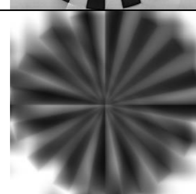
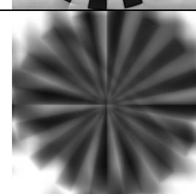
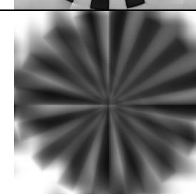
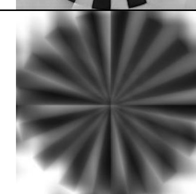
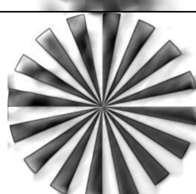
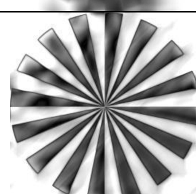
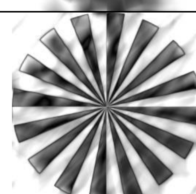
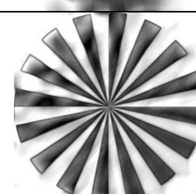
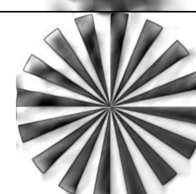
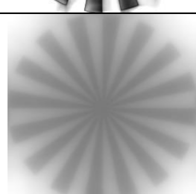
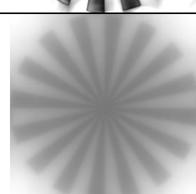
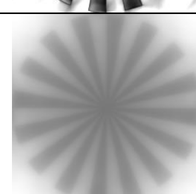
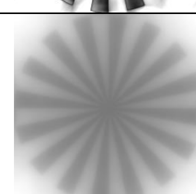
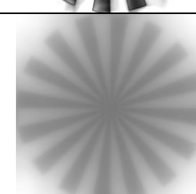
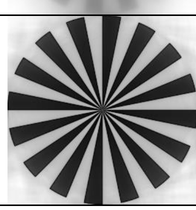
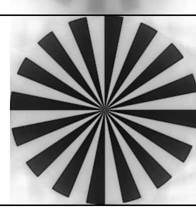
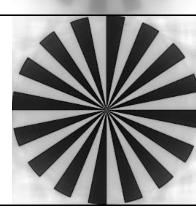
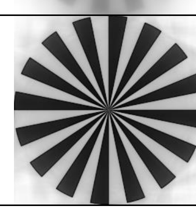
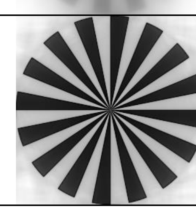
Figure 3: Defocused *MTFs* corresponding to different defocus parameters. The invariance can be seen in the defocus range

4. RESULTS AND CONCLUSIONS

A set of simulations has been prepared to illustrate the behavior of the preceding equations. The defocus invariant imaging system considered is a single meniscus lens and the phase-masks described above with the nominal values presented in the Table 1.

Table 3 shows the numerical results of a simulated WFC system using different phase-masks. The W_{20} parameter is related to defocus. The second row in Table 3 shows the images that would be obtained from a conventional optical system It can be seen a blurred image when amount of defocus increases. The numerical results shows defocus invariance that ranges from $\left[-5\lambda, 5\lambda\right]$ in *WFC* system. Decoded images from the cubic *PM* and the sum of 4 Zernike polynomials *PM* are significantly degraded by image artifacts. Decoded images resulting from the trefoil *PM* show minimal artifacts and the contrast is better. So the *WFC* system works properly due to the proposed optimal phase deviation parameter α using a non-exhaustive search algorithm.

Table 3: Simulation of the image formation process through an optical system. Second row conventional optical system.

Mask	$W_{20} = -5\lambda$	$W_{20} = -3\lambda$	$W_{20} = 0\lambda$	$W_{20} = 3\lambda$	$W_{20} = 5\lambda$
$\boldsymbol{\alpha = 0\lambda}$					
Sum. 4 Zernike polynomials $\boldsymbol{\alpha = 19\lambda}$					
Sum. 4 Zernike polynomials $\boldsymbol{\alpha = 19\lambda}$ *decoded*					
Cubic-phase $\boldsymbol{\alpha = 20\lambda}$					
Cubic-phase $\boldsymbol{\alpha = 20\lambda}$ *decoded*					
Trefoil $\boldsymbol{\alpha = 18\lambda}$					
Trefoil $\boldsymbol{\alpha = 18\lambda}$ *decoded*					

ACKNOWLEDGEMENTS

E. Gonzalez-Amador would like to thank the National Council for Science and Technology (CONACyT) for the scholarship number 423645.

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