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**SPIE.**

# Optical spherometer for measuring large curvature radii of convex surfaces

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## ABSTRACT

Measuring large curvature radii of convex surfaces with high precision is a challenge because the spherometer's focus must be positioned at the apex of the surface and at the center of curvature of the surface by moving the surface or the spherometer. If the radius of curvature is larger than the back focus of spherometer, then measurement is not possible. In this work, we propose to use the FOCOIVA system<sup>1</sup> to move the focus of the spherometer in longitudinal way without modifying the  $f$  number by moving two lenses inside it, with this mechanism it is possible to measure radii of curvature of several meters in length. The curves of movement of the lenses and the optical parameters of the lenses that compose the spherometer are presented.

**Keywords:** Optical design, spherometer, radii of curvature, convex surfaces.

## 1. INTRODUCTION

Actually, in modern optics, the design and manufacture of lenses with high quality convex surfaces are more requested due to the precision of the optical systems for their use and as a consequence the better results obtained in different research areas and in the industry. To determine whether the manufactured spherical optical elements conform to design specifications and optical tolerance levels, lenses manufacturers must have adequate measuring instruments, evaluate their uncertainty, or rely on reliable radius of curvature inspections by certified institutes to ensure that lenses are adjusted to the accuracy of requirements<sup>2</sup>.

The commonly used instruments that allow us to measure the radius of curvature of a surface or area of interest that is being manufactured are known as spherometers<sup>3</sup> and are classified as mechanical, optical or interferometric. In order to determine what type of spherometer we can use it is necessary to take into account at what stage of manufacture it is desired to measure the radii of curvature and the shape of the surface<sup>4</sup>. For example, mechanical type spherometer are used in the initial stages of the manufacturing process, while optical or interferometric are generally used in the polishing stages and to measure smooth surfaces with large radii of curvature<sup>5 6</sup>. After a lens is polished, the radius of curvature must be examined to ensure and control the output quality. Because of the radius of curvature errors of the lens, they affect the optical quality of the optical systems. Designers should evaluate the bend radius error tolerance and create reference specifications to follow for optical lens manufacturers. The variation of the radius of curvature of the lens should be constantly examined when the curved surface of a lens is formed and in the polishing processes to facilitate the timely adjustment of the parameters<sup>7</sup>.

## 2. FIRST ORDER ANALYSIS OF FOCOIVA SYTEM

When we measure radii of curvature with a traditional spherometer, we first place the surface in the position where the spherometer's focus coincides with the apex of the surface, then the surface or the spherometer moves until the spherometer's focus coincides with the center of curvature of the surface, the distance traveled in this procedure corresponds to the value of the radius of curvature.

In the case of convex surfaces, the radii of curvature that can be measured must be smaller than the back focal length of the spherometer because the surface cannot physically pass through the spherometer.

To overcome the range of radii values of curvature of convex surfaces that can be measured, we propose to use an optical system called FOCOIVA<sup>1</sup> to move the focus of the spherometer along the optical axis without changing the number  $f$  of the light beam and therefore, maintaining the same resolution during the displacement of the focus. With

this optical system, it is not necessary to move the spherometer or the surface during the measurement process, therefore, it will be possible to measure radii of curvature longer than those that can be measured with a traditional spherometer.

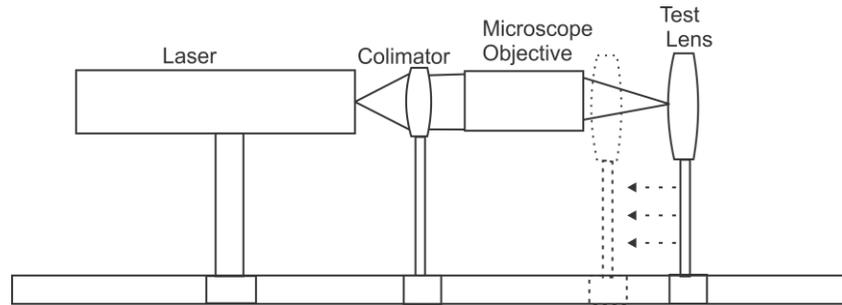


Figure 1. Traditional Optical Spherometer.

In any optical system, by moving one of its lenses we change the position of the image plane and also vary the effective focal length. However, in the FOCOIVA system, the main characteristic is to keep the effective focal length constant and to vary the distance to image plane, unlike to zoom systems that keep the image plane distance constant and change the effective focal length. To perform this operation, two lenses must be moved simultaneously and coordinately following different movement curves, whereby one of them varies the position of the image while the other maintains constant the effective focal length.

The design of the proposed FOCOIVA system will allow the measurement of convex surfaces of radius of curvature of at least 1000 mm without having to move the convex test surface. This optical system consists of an optical arrangement composed of four lenses, with a (- + - +) configuration although the configuration may vary depending on the parameters of the optical arrangement.

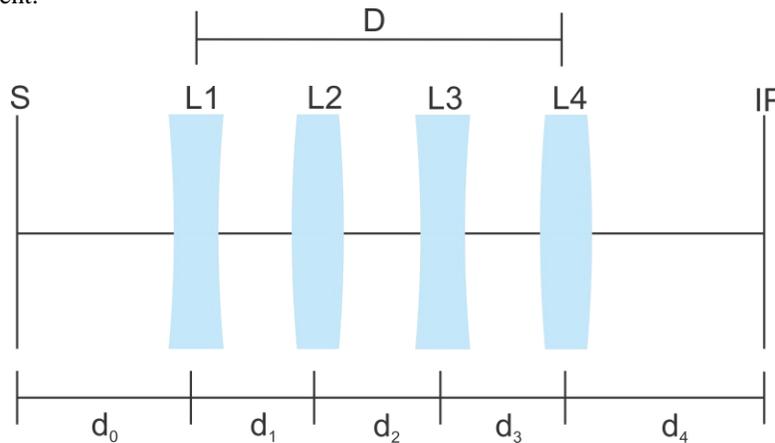


Figure 2. FOCOIVA system.

In this system S is the position of the entrance pupil, L1 and L4 are fixed lenses, while L2 and L3 are motion lenses, IP is the image plane and D is the size of the optical arrangement. To perform the first-order analysis of the FOCOIVA system we use the matrix method<sup>8,9</sup> to calculate the total matrix of elements.

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & -\varphi_4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ d_3 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\varphi_3 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ d_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\varphi_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ d_1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -\varphi_1 \\ 0 & 1 \end{bmatrix} \quad (3)$$

where the  $\varphi_i$  are the lens optical powers y the  $d_i$  are distances between lenses.

So, the elements of matrix  $M$  are

$$a = (1 - d_1\varphi_2) \left[ (1 - d_3\varphi_4)(1 - d_2\varphi_3) - d_2\varphi_4 \right] - d_1 \left[ (1 - d_3\varphi_4)\varphi_3 + \varphi_4 \right], \quad (4)$$

$$b = - \left[ (1 - d_1\varphi_2)\varphi_1 + \varphi_2 \right] \left[ (1 - d_3\varphi_4)(1 - d_2\varphi_3) - d_2\varphi_4 \right] - (1 - d_1\varphi_1) \left[ (1 - d_3\varphi_4)\varphi_3 + \varphi_4 \right], \quad (5)$$

$$c = (1 - d_1\varphi_2) \left[ (1 - d_2\varphi_3)d_3 + d_2 \right] + d_1(1 - d_3\varphi_3), \quad (6)$$

$$d = (1 - d_1\varphi_1)(1 - d_3\varphi_3) - \left[ (1 - d_1\varphi_2)\varphi_1 + \varphi_2 \right] \left[ (1 - d_2\varphi_3)d_3 + d_2 \right], \quad (7)$$

where the total optical power  $\varphi$  is

$$\varphi = -b, \quad (8)$$

substituting equation (5) in equation (8), we have

$$\begin{aligned} \varphi = & - \left[ (1 - d_1\varphi_2)\varphi_1 + \varphi_2 \right] \left[ (1 - d_3\varphi_4)(1 - d_2\varphi_3) - d_2\varphi_4 \right] + (1 - d_1\varphi_1) \\ & \times \left[ (1 - d_3\varphi_4)\varphi_3 + \varphi_4 \right]. \end{aligned} \quad (9)$$

If the object is in infinity, the equation to marginal ray is

$$\begin{bmatrix} u_4' \\ h_4 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 \\ h_0 \end{bmatrix} \quad (10)$$

where  $u_4'$  is the angle of convergence in image space,  $h_0$  is the height of incident ray in lens L1 and  $h_4$  is the height of incident ray in lens L4.

Then

$$u_4' = bh_0, \quad (11a)$$

$$h_4 = dh_0, \quad (11b)$$

and we have

$$d_4 = - \left( \frac{h_4}{u_4'} \right). \quad (12)$$

Therefore, by substituting equations (11a), (11b), (8) and (7) we have

$$d_4 = \left( \frac{1}{\varphi} \right) \left\{ (1 - d_1\varphi_1)(1 - d_3\varphi_3) - \left[ (1 - d_1\varphi_2)\varphi_1 + \varphi_2 \right] \left[ (1 - d_2\varphi_3)d_3 + d_2 \right] \right\}. \quad (13)$$

As  $D$  is the size of the optical arrangement, then

$$D = d_1 + d_2 + d_3. \quad (14)$$

The following equation is for the principal ray,

$$\begin{bmatrix} \bar{u}_4' \\ \bar{h}_4 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \alpha \\ \alpha d_0 \end{bmatrix}, \quad (15)$$

where  $\alpha$  is the semi-angular field of view.

Therefore, the angular magnification is

$$m_\alpha = \frac{\bar{u}_4'}{\alpha}, \quad (16)$$

And substituting equations (15), (4), (5), (6) and (7) in (15)

$$m_\alpha = (1 - d_1\varphi_2) \left[ (1 - d_3\varphi_4)(1 - d_2\varphi_3) - d_2\varphi_4 \right] - d_1 \left[ (1 - d_3\varphi_4)\varphi_3 + \varphi_4 \right] - \varphi d_0. \quad (17)$$

In the FOCOIVA optical system there are mainly two cases for lens movement<sup>1</sup>; case 1 where the lenses move in opposite directions (from the ends to the center) and case 2, where the lenses move in the same direction (from left to right).

## 2.1 Lenses moves in opposite directions

The initial conditions are

$$d_{1i} = 0, \quad (18a)$$

$$d_{2i} = D, \quad (18b)$$

$$d_{3i} = 0. \quad (18c)$$

Equation (19) describes the movement of the lenses in opposite directions

$$\begin{aligned} & -\varphi_3\varphi_4 \left[ (1 - d_1\varphi_2)\varphi_1 + \varphi_2 \right] d_3^2 + \left\{ \left[ 1 + \varphi_4(D - d_1) \right] \left[ (1 - d_1\varphi_2)\varphi_1 + \varphi_2 \right] - \varphi_4(1 - d_1) \right\} \varphi_3 d_3 + \\ & \left\{ \left[ (1 - d_1\varphi_2)\varphi_1 + \varphi_2 \right] \left[ 1 - (D - d_1)(\varphi_3 + \varphi_4) \right] + (1 - d_1\varphi_1)(\varphi_3 + \varphi_4) - \varphi \right\} = 0. \end{aligned} \quad (19)$$

The total power of the system, the initial distance  $d_{4i}$  and the angular amplification are defined by equations (20), and (21).

$$\varphi = \varphi_1 + \varphi_2 + \varphi_3 + \varphi_4 - D(\varphi_1 + \varphi_2)(\varphi_3 + \varphi_4), \quad (20)$$

$$d_{4i} = \left( \frac{1}{\varphi} \right) \left[ 1 - D(\varphi_1 + \varphi_2) \right], \quad (21)$$

To calculate the powers of the L1, L3 and L4 lenses we use the equations (23), (24) and (25)

$$\varphi_1 = C_2 - \varphi_2, \quad (22)$$

$$\varphi_3 = C_1 - \varphi_4, \quad (23)$$

$$\varphi_4 = \frac{C_2}{1 - DC_2}, \quad (24)$$

Where  $C_1$  and  $C_2$  are defined by

$$C_1 = \frac{1 - m_{\alpha i} - \varphi d_0}{D}, \quad (25a)$$

$$C_2 = \frac{1 - \varphi d_{4i}}{D}. \quad (25b)$$

Then, the angular amplification is

$$m_{\alpha i} = \frac{1 - \varphi D}{1 - C_2 D} - \varphi d_0. \quad (26)$$

Therefore, we have only two independent variables  $d_1$  and  $\varphi_2$ .

## 2.2 Lenses moves in the same direction (left to right)

The initial conditions are

$$d_{1i} = 0, \quad (27a)$$

$$d_{2i} = 0, \quad (27b)$$

$$d_{3i} = D. \quad (27c)$$

For this case, when the lenses move in the same direction, we use the equation (28)

$$\begin{aligned} & -\varphi_3 \varphi_4 \left[ (1 - d_1 \varphi_2) \varphi_1 + \varphi_2 \right] d_2^2 + \left\{ \left[ (1 - d_1 \varphi_2) \varphi_1 + \varphi_2 \right] \left[ -1 + (D - d_1) \varphi_4 \right] + (1 - d_1 \varphi_1) \varphi_4 \right\} \varphi_3 d_2 \\ & + \left\{ \left[ (1 - d_1 \varphi_2) \varphi_1 + \varphi_2 \right] \left[ 1 - (D - d_1) \varphi_4 \right] + (1 - d_1 \varphi_1) \left[ \varphi_3 + \varphi_4 - (D - d_1) \varphi_3 \varphi_4 \right] - \varphi \right\} = 0. \end{aligned} \quad (28)$$

To calculate the powers of the L1, L3 and L4 lenses we use the equations (29), (30) and (31)

$$\varphi_1 = \frac{C_1}{1 - C_1 D} - \varphi_2, \quad (29)$$

$$\varphi_3 = \frac{C_2 (1 - C_1 D) - C_1}{1 - C_1 D}. \quad (30)$$

$$\varphi_4 = \frac{1 - m_{\alpha} - \varphi d_0}{D}, \quad (31)$$

Thus

$$\varphi_4 = C_1. \quad (32)$$

For this case, we also have two independent variables  $\varphi_2$  y  $d_1$ .

### 3. COMPUTER OF PARAXIAL PARAMETERS

For the design of the optical spherometer, we use a Gaussian beam equation to calculate the effective focal length of the system<sup>10</sup>

$$\left(w_0^2 - w_0'^2\right) f^2 + \left(2z w_0'^2\right) f - \left(w_0'^2 z^2 + \frac{\pi^2 w_0^4 w_0'^2}{16 \lambda^2}\right) = 0, \quad (33)$$

where  $w_0$  y  $w_0'$  are the diameters of the input and output beam waist,  $z$  is the width of the input beam waist,  $\lambda$  is the wavelength, and  $f$  is the effective focal length. We use  $\lambda = 543$  nm of He-Ne laser,  $w_0 = 5.217$  mm,  $z = 30$  mm. Solving the quadratic equation, we have  $f = 603.7358$  mm.

For the case 1, when the lenses move in opposite directions, We defined  $d_1$  in a range of [0,132.7] mm and the lens power for L2  $\varphi_2 = 0.0084$  with which we ensure that the discriminant is  $\delta \geq 0$ , in addition to a total size of the array  $D = 150$  mm, the initial image distance  $d_{4i} = 180$  mm and the distance of S to L1  $d_0 = 50$  mm.

Solving the equation (19) we obtain a range of motion of the focus of the spherometer of 1026.476 mm when  $d_1 = 132.700$  mm,  $d_2 = 0.2420$  mm,  $d_3 = 17.05794$  mm and  $d_4 = 1192.252$  mm. The MATLAB software was used, where the equations of motion of FOCOIVA system were programmed and the motion curves were plotted, which are shown in Figure 3.

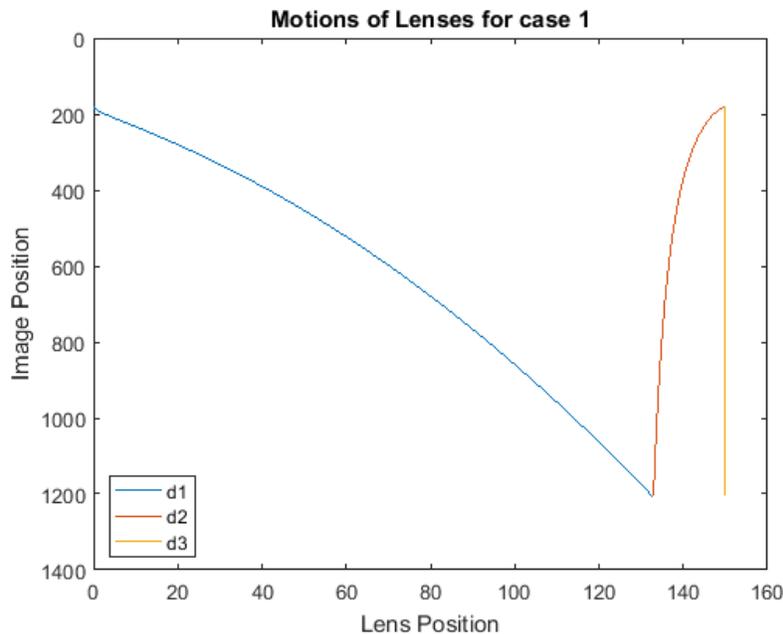


Figure 3. Motion curves of lenses for case 1.

We then analyzed the calculated data (motion lenses and image distance) using the ZEMAX software, paraxial lenses were used for the powers and the multi-configuration tool<sup>11</sup> for the different distances. Which are shown in Table 1.

Configurations	$d_1$	$d_2$	$d_3$	$d_4$
1	2.000000000	145.6497841	2.350215940	196.1703147
2	15.000000000	128.6946754	6.305324556	256.5350522
3	30.000000000	110.9857319	9.014268139	332.8492180
4	45.000000000	93.97709678	11.02290322	421.4298438
5	60.000000000	77.40392609	12.59607391	523.0231349
6	75.000000000	61.13616498	13.86383502	637.9281282
7	90.000000000	45.09355595	14.90644405	766.3154816
8	105.0000000	29.22211621	15.77788379	908.2975606
9	120.0000000	13.48380961	16.51619039	1063.953749
10	131.9000000	1.074214076	17.02578592	1197.207382

Table 1. Multi configurations in ZEMAX.

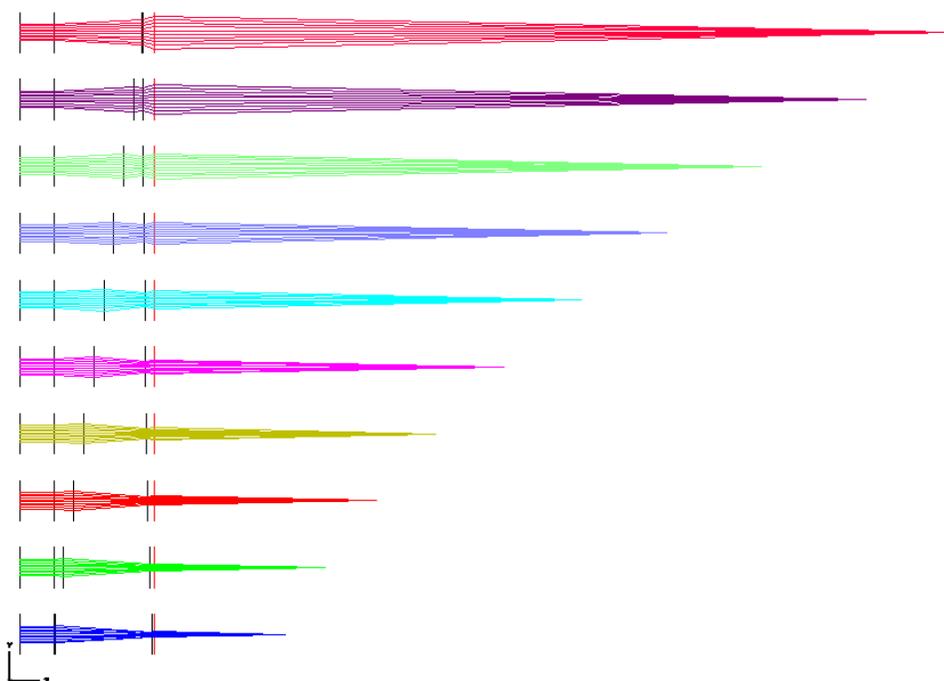


Figure 4. Layout of the multi configurations in ZEMAX, for the different positions of the lenses L2 and L3.

In table 1 and Figure 4 we observe that, for the different positions of the lenses L2 and L3, the image distance changes, while L1 and L4 are kept fixed. We also observe that the focus of the system moves as the lenses move without changing the overall size of the system. In addition, we can observe the shape of the movement curves previously calculated with MATLAB.

#### 4. FINAL DESIGN

In this section, we design thick lenses to replace the paraxial lenses and therefore we have a real design to the FOCOIVA system which is the basis of the optical spherometer. For this, we need to find the appropriate construction parameters to obtain the best performance of the optical system. We use the ZEMAX optical design software to perform system optimization.

For the optimization of the system we use the local optimization tool<sup>12</sup>, defining a merit function<sup>13</sup>. For this design of the FOCOIVA system, doublet lenses were used<sup>14</sup>, the characteristics are presented in Table 2.

Parameters	L1	L2	L3	L4
Ratio of curvature 1	65.55671	-1667.577	-52.36769	122.4657
Ratio of curvature 2	24.11944	-1421.986	-44.94974	80.45711
Ratio of curvature 3	36.96876	-78.32792	71.41547	-89.95616
Thickness 1	4.5	5	5	4.5
Thickness 2	4.5	4	4	8
Material	BK7/SF2	BK7/SF2	BK7/SF2	BK7/SF2
Diameter	20	20	20	20

Table 2. Characteristics of doublet lenses.

Figure 5 shows the layout of the system where you can see the shape of each of the double lenses.

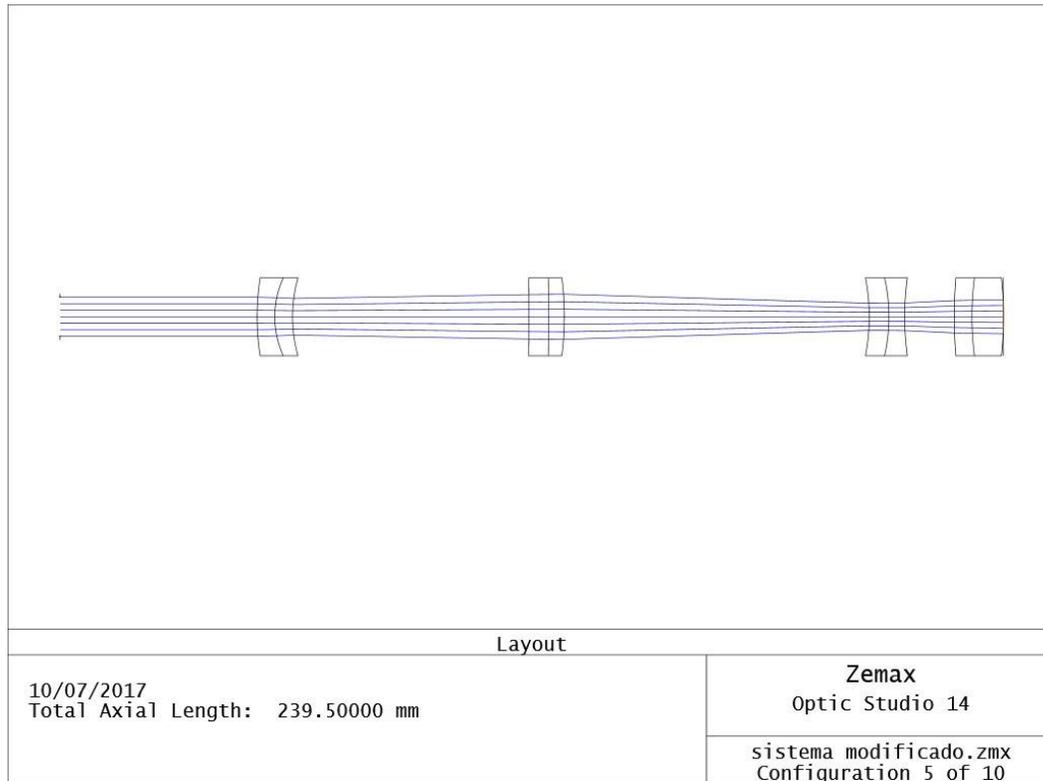


Figure 5. Layout of FOCOIVA System using doublet lenses.

We also present the layout for the different configurations in Figure 6, using doublet lenses, in order to visualize the different positions of the lenses in the FOCOIVA system.

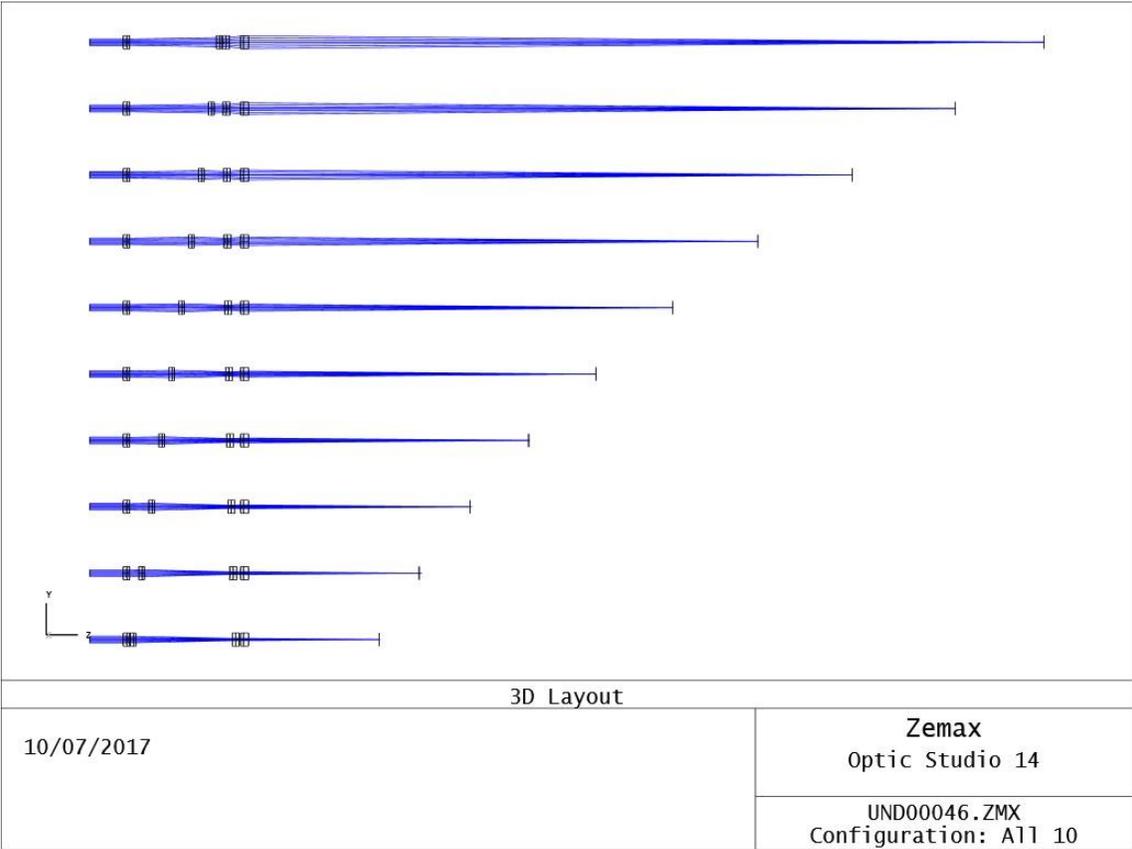


Figure 6. Layout of the multi configurations in ZEMAX, for the different positions of the lenses L2 and L3 using doublet lenses.

In Figure 6 we can be seen that the lenses L1 and L4 are fixed, whereas L2 and L3 move in opposite directions, which allows the image distance in each of the configurations to be varied, while maintaining the effective focal length.

To do the analysis of the System we use the tool of Configuration Matrix, with it we can observe the diagrams of spots corresponding to each configuration as shown in Figure 7.

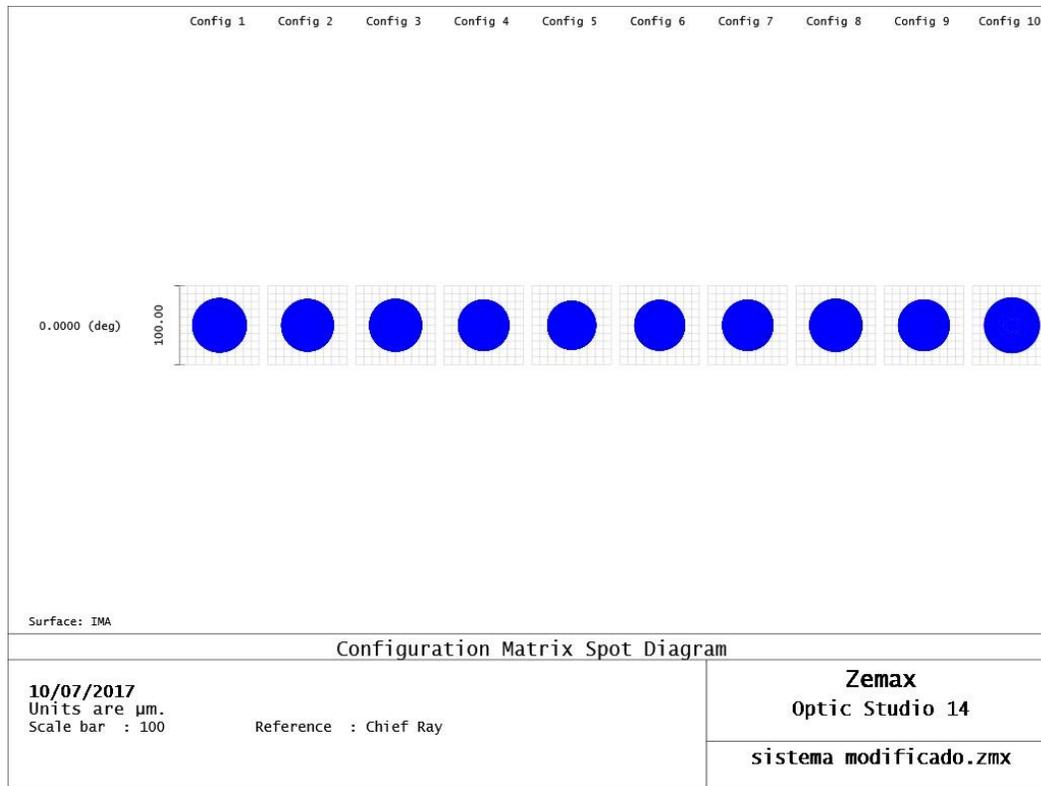


Figure 7. Spot diagrams for all configurations of FOCOIVA System.

In Figure 7 it can be seen that the size of the plot of stains is constant in the different positions of the system, which ensures that the variation of the image distance is adequate to the parameters of the system.

## 5. CONCLUSIONS

The results obtained using the optical system FOCOIVA for the design of an optical spherometer demonstrate that with this system it is possible to measure radii of curvature of convex surfaces having a maximum radius of curvature of 1000 mm.

It was shown that the size of the spots in the focus of each configuration is practically the same and therefore the resolution in the radius of curvature measurement is invariant for radii of curvature of 10 mm to 1000 mm.

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